8. M. J. McCroskey and J. G. McDougall, "Shock wave shapes on a sharp flat plate in rarefied hypersonic flow," AIAA J., 4, 184 (1966).
9. D. E. Rothe, "Flow visualization using a traversing electron beam," AIAA J., 3, 1945 (1965).
10. R. E. Hoglund, "Recent advances in gas-particle nozzle flows," ARS, 32, 662 (1962).
11. A. B. Bauer, "Normal shock location of underexpanded gas-particle jet," AIAA J., 3, 1187 (1965).
12. C. H. Lewis and D. J. Carlson, "Normal shock location of underexpanded gas and gas-particle jets," AIAA J., 2, 776 (1964).
13. V. I. Blagosklonov and A. L. Stasenko, "Two-dimensional flows of a multiphase mixture in a nozzle and jet exhausting into a flooded space," Uch. Zap. TsAGI, 8, No. 1, 32 (1977).
14. S. A. Senkovenko, "Structure of an underexpanded supersonic CO_2 jet," Proc. of Second Republican Conference on Aerodynamics and Heat and Mass Transfer, Section: Aerodynamics at High Velocities [in Russian], Kiev (1971), pp. 154-160.
15. G. V. Tsiklauri, V. S. Danilin, and L. I. Seleznev, Adiabatic Two-Phase Flows [in Russian], Atomizdat, Moscow (1973).

EXTERNAL EXCHANGE IN A DISPERSE BED

Yu. A. Buevich and E. B. Perminov

UDC 536.242:532.546

The time-varying convective heat or mass transfer from objects immersed in a filtering granular bed is analyzed. Absorption of the heat or mass in the bed is taken into account.

Heat and mass transfer between a bed and the surface of objects immersed in the bed is important in many processes involving heat treatment or diffusion treatment of articles in still or fluidized granular beds, during the cooling of furnaces and reactors with granular beds caused by inserting special heat exchangers into them, etc.

If the characteristic linear dimension of the immersed object is much larger than the characteristic dimension of the microstructure of the bed (e.g., the grain diameter) it is natural to use the continuum transport equations in the various phases in the bed to describe these processes. In several important cases these two equations can be replaced by a single transport equation; this approach corresponds to adopting a model for the disperse system around the object consisting of a homogeneous continuous medium which is described by appropriate effective thermal and diffusion properties. It is important to note that in general the effective thermal diffusivity and diffusion coefficient, which represent not only molecular transport but also the transport due to the convective heat and mass dispersion in the discontinuous pore space of the filtering granular bed, are inhomogeneous and depend on the local filtration velocity of the continuous phase in the bed.

The problems of the steady-state convective transfer from objects in a granular bed penetrated by a filtering flow were first formulated and solved in [1, 2], where absorption of heat or mass was neglected. Absorption has been taken into account in these problems on the basis of the film or penetration theory, as a rule; i.e., absorption has been taken into account by ignoring convection, in accordance with mass-transfer systems in which chemical reactions are occurring (see, e.g., the review in [3, 4]). In [5] there is an example of the application of a penetration theory to the analysis of external heat transfer in a fluidized bed. In the first case, as is usual in convective-diffusion processes, the surface in the flow is far from uniformly accessible with respect to diffusion; in the second case, it is uniformly accessible.

In actual granular-bed installations it is extremely common to find situations in which the convective transport and the acceleration of this transport caused by the absorption of

Institute of Problems of Mechanics, Academy of Sciences of the USSR, Moscow. Tambov Institute of Chemical Engineering. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 2, pp. 254-263, February, 1981. Original article submitted December 20, 1979.

 0022-0841/81/4002-0150$07.50

a substance within the bed are equally important. The impurity diffusing through the bed may be absorbed as the result of homogeneous or heterogeneous chemical reactions; heat may be absorbed by particles which are rapidly removed from the heat-exchange surface in a state of pronounced fluidization [5]; and heat may also be absorbed, e.g., because of a high heat of reaction of catalytic reactions which occur in reactors with a bed of catalyst particles [6]. It is thus necessary to consider both convective diffusion and the absorption of heat or mass in the granular bed. We need a better picture of events than is given us by the comparatively few theoretical papers on convection complicated by chemical reactions [4].

Below we will discuss this problem in the time-varying, single-phase case, and we will solve the problem for the particular cases of a plate oriented parallel to the flow, of a circular cylinder with axis normal to the flow, and of a sphere at high Peclet numbers.

Formulation of the Problem. We consider the convective-diffusion equation in the form

$$\alpha\frac{\partial T}{\partial t}+(\mathbf{u}\nabla)T=\nabla(\mathbf{D}_e\nabla T)-kT, \tag{1}$$

where

$$\alpha=\varepsilon+(1-\varepsilon)\frac{c_1d_1}{c_0d_0},\quad \mathbf{D}_e=\frac{\lambda_e}{c_0d_0},\quad k=\frac{K}{c_0d_0}. \tag{2}$$

The equation corresponds to a single-temperature model for the dispersive medium, which is a good approximation if the time variation of the transport process is slow, if the contact conductivity of the particles can be ignored, and if the transport caused by the average and fluctuating motion of the disperse phase can be ignored [7]. The same equation describes external heat transfer in the opposite limit — in a bed with pronounced fluidization, in which case there is a rapid mixing of the disperse phase. In other words, in this case the exchange of particles, which serve as heat sinks, between the core of the bed and the surface zone is very rapid [5], and we have $\alpha = \varepsilon$. In problems involving the diffusion in the gaps between particles, we must assume $c_0d_0 = 1$ and $c_1d_1 = 0$.

The tensor of the effective thermal diffusivity or effective diffusion coefficient incorporates both molecular transport and convective dispersion. Its eigenvalues can be written [1, 2, 8]

$$D_{e,1}=D_{\parallel}=D+2k_{\parallel}au,\ D_{e,2}=D_{e,3}=D_{\perp}=D+2k_{\perp}au. \tag{3}$$

For definiteness we will use the coefficients $k_{\parallel} = 0.76$ and $k_{\perp} = 0.19$ calculated in [8]. We direct the x_1 axis along the local filtration velocity u. This velocity is assumed to be independent of the time and known, e.g., from a solution of the corresponding filtration problem. The effective coefficient D is also assumed to be a known function of the physical properties of the bed.

We write the initial and boundary conditions for the solution of (1) in the form

$$T|_{t=0}=0,\ T|_{r\to\infty}=0,\ T+\varkappa\frac{\partial T}{\partial n}\bigg|_{\mathbf{r}\in S}=T_0(S), \tag{4}$$

where S is the surface area of the object, and T_0 may in general depend on the position of the point on this surface.

The solution of problem (1), (4) can be reduced to an integration of the solution of the corresponding problem with k = 0, i.e., without absorption [9]. Examining the solution of the problem which follows from (1), (4) after Laplace time transforms are taken, we can easily show that

$$T=\frac{k}{\alpha}\int\limits_0^t T^\circ\exp\left(-\frac{kt}{\alpha}\right)dt+T^\circ\exp\left(-\frac{kt}{\alpha}\right), \tag{5}$$

$$T=T(t,\ \mathbf{r};\ k),\ T^\circ=T(t,\ \mathbf{r};\ 0).$$

The solution of the steady-state problem corresponding to (1), (4) is found from (5) in the limit $t \to \infty$. Our first task is thus to find the solution of the time-varying problem with k = 0. An equation like (5) is also valid for any linear function of T and of its derivatives with respect to the coordinates, in particular, for the flux of a substance away from the surface of an object.

In the particular problems discussed below we will assume that the "macroscopic" Peclet number Pe, which is a measure of the convective diffusion toward the object in the flow, is large. Then T is nonzero only in a thin region near the surface, over whose thickness T varies much more rapidly than along the surface itself. In this case we can approximate the components of **u** by their values in the immediate vicinity of the surface, and in the first term on the right side of (1) we can ignore the derivatives with respect to the tangential coordinates in comparison with the derivatives along the normal. Steady-state problems with k = 0 can be solved by reducing (1) to the ordinary heat-conduction equation [10], which is a particular case of the Poincare–Lighthill–Ho method; an alternative approach is to use the latter method in the more general form, as in [1, 2].

It is not sufficient here to solve the time-varying problems with k = 0. Below we will consider only problems with a boundary condition of the first kind at the surface of the object [$\varkappa = 0$ in (4)] with T_0 = const, in which case we can introduce a self-similar variable and reduce the problem to that of solving an ordinary differential equation and a first-order partial differential equation, as proposed by Ruckenstein [11].

Plate. We can illustrate the general method for the problem of the exchange of a bed with a plate oriented parallel to a filtration flow. This problem can also be solved exactly; the exact solution holds for arbitrary values of Pe and k [12] but is extremely complicated. In this case we have

$$\mathbf{u} = \text{const},\ D_\perp = (1+\gamma) D,\ \gamma = 0.38au/D, \tag{6}$$

and the coefficient γ is a "microscopic" (structural) Peclet number for the particles of the granular bed.

From (1), (6) we have the equation

$$\alpha \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = (1+\gamma) D \frac{\partial^2 T}{\partial y^2} - kT, \tag{7}$$

which must be solved under the conditions

$$T|_{t=0} = 0,\ T|_{y\to\infty} = 0,\ T|_{y=0} = T_0. \tag{8}$$

Using the approximation of a thin diffusion bed with k = 0, we set $T^\circ = T^\circ(\eta)$, where $\eta = y/\delta(t, x)$. Then from (7) we find

$$\frac{d^2T^\circ}{d\eta^2} + \eta \frac{dT^\circ}{d\eta}\left\{\frac{1}{2(1+\gamma)D}\left(\alpha \frac{\partial\varphi}{\partial t} + u \frac{\partial \varphi}{\partial x}\right)\right\} = 0, \tag{9}$$

where $\varphi = \delta^2$. The expression in braces in (9) must be equal to a constant; it is easy to see that the function $T^\circ(\eta)$ is invariant with respect to the choice of this constant. If we set it equal to two, then Eq. (9) splits into the two equations

$$\frac{d^2T}{d\eta^2} + 2\eta \frac{dT}{d\eta} = 0,\ \alpha \frac{\partial\varphi}{\partial t} + u \frac{\partial\varphi}{\partial x} = 4(1+\gamma) D. \tag{10}$$

The solutions of these equations under conditions (8) and under the obvious condition $\varphi = 0$ at t = 0 are of the form

$$T^\circ = T_0 \operatorname{erfc} \eta = T_0 \operatorname{erfc}[y/\delta(t, x)], \tag{11}$$

$$\varphi = \delta^2 = \begin{cases} 4(1+\gamma)(D/\alpha)t, & t < (\alpha/u)x, \\ 4(1+\gamma)(D/u)x, & t > (\alpha/u)x. \end{cases} \tag{12}$$

In the derivation of (11) and (12) we actually made no assumption regarding Pe; these equations thus hold for an arbitrary value of Pe if the longitudinal dispersive transport of the substance is negligible in comparison with the convective transport.

The local flux of the substance away from the plate in the case k = 0 is

$$q^\circ = -(1+\gamma) D \left.\frac{\partial T^\circ}{\partial y}\right|_{y=0} = \begin{cases} T_0[\alpha(1+\gamma) D/\pi t]^{1/2}, & t < (\alpha/u)x, \\ T_0[u(1+\gamma) D/\pi x]^{1/2}, & t > (\alpha/u)x. \end{cases} \tag{13}$$

At a point at a distance x from the front edge of the plate a steady state is thus reached after a finite time $(\alpha/u)x$.

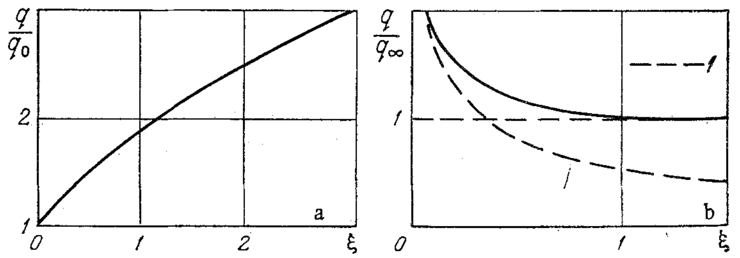

Fig. 1. a) Ratio of the local flux away from the plate to its value in the limit $k \to 0$ (a) as a function of the dimensionless longitudinal coordinate ξ; b) the same, in the case $k \to \infty$. The horizontal dashed line and the dashed curve correspond to the limiting cases $q/q_\infty = 1$ and $q/q_\infty = 1/\sqrt{\pi\xi}$.

The solution of the corresponding problem with $k \neq 0$ is found from (11)-(13) with the help of general equation (5). In particular, we have

$$q_0 = \begin{cases} T_0\left\{\sqrt{(1+\gamma)Dk}\,\mathrm{erfc}\sqrt{\frac{kt}{\alpha}} + \sqrt{\frac{\alpha(1+\gamma)D}{\pi t}}\exp\left(-\frac{kt}{\alpha}\right)\right\}, & t < \frac{\alpha x}{u}, \\ T_0\left\{\sqrt{(1+\gamma)Dk}\,\mathrm{erfc}\sqrt{\frac{kx}{u}} + \sqrt{\frac{u(1+\gamma)D}{\pi x}}\exp\left(-\frac{kx}{u}\right)\right\}, & t > \frac{\alpha x}{u}. \end{cases} \tag{14}$$

For small and large values of kx/u we find

$$q \approx \begin{cases} T_0\sqrt{\frac{u(1+\gamma)D}{\pi x}}\left(1 + \frac{kx}{u}\right), & \frac{kx}{u} \ll 1, \\ T_0\sqrt{(1+\gamma)Dk} + o\left\{T_0\frac{u(1+\gamma)D}{\pi x}\exp\left(-\frac{kx}{u}\right)\right\}, & \frac{kx}{u} \gg 1. \end{cases} \tag{15}$$

The first of these expressions corresponds to the case in which the absorption of the substance is slight; the second expression corresponds to the case in which this absorption is pronounced, and the surface far from the front edge of plate becomes uniformly accessible in practice. The corresponding equations for the steady state ($t \to \infty$) are particularly important. In this case we find from (14)

$$\frac{q}{q_0} = \exp(-\xi) + \sqrt{\pi\xi}\,\mathrm{erfc}\sqrt{\xi},\quad \frac{q}{q_\infty} = \mathrm{erf}\sqrt{\xi} + \frac{1}{\sqrt{\pi\xi}}\exp(-\xi),$$
$$q_0 = T_0\sqrt{\frac{(1+\gamma)Du}{\pi x}},\quad q_\infty = T_0\sqrt{(1+\gamma)Dk},\quad \xi = \frac{kx}{u}. \tag{16}$$

Figure 1 shows the ratios in (16) as functions of the dimensionless longitudinal coordinate. In particular, we see that the absorption of the substance corresponds to the establishment of a uniform local flux over the surface of the plate. At small values of x the substance is removed primarily by convection, while at large values of x it is removed primarily by absorption.

For the total flux of the substance away from a plate of length L we find from (16)

$$\frac{Q}{Q_0} = \frac{1}{2}\left[\exp(-\xi_L) + \sqrt{\pi}\left(\sqrt{\xi_L} + \frac{1}{2\sqrt{\xi_L}}\right)\mathrm{erf}\sqrt{\xi_L}\right],$$
$$\frac{Q}{Q_\infty} = \left(1 + \frac{1}{2\xi_L}\right)\mathrm{erf}\sqrt{\xi_L} + \frac{1}{\sqrt{\pi\xi_L}}\exp(-\xi_L), \tag{17}$$
$$Q_0 = 2T_0\sqrt{\frac{(1+\gamma)DuL}{\pi}},\quad Q_\infty = T_0\sqrt{(1+\gamma)Dk}\,L,\quad \xi_L = \frac{kL}{u}.$$

These results are plotted in Fig. 2. We see in particular that absorption increases the flux considerably.

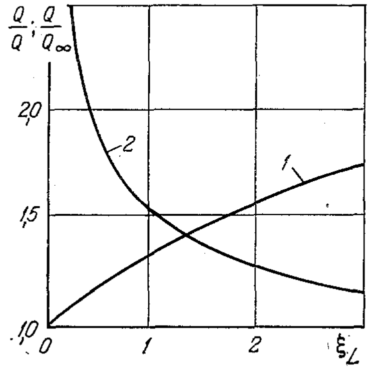

Fig. 2. 1) Ratio of the total flux away from the plate to its value in the limit k → 0 as a function of the dimensionless length of the plate ξ_L; 2) the same, for the limit k → ∞.

Under otherwise equal conditions, an increase in the radius of the particles in the bed, a, increases the structural Peclet number γ and thus the total flux (at $\gamma \gg 1$ we have $Q \sim \sqrt{a}$). An increase in the filtration velocity u also increases γ, but it simultaneously reduces the parameter ξ_L. It is easy to see that the increase in Q with increasing u is more pronounced, the shorter the plate.

The results given above also hold for an arbitrary cylindrical surface with rectilinear generator directed along the flux, provided that δ, the thickness of the wall layer, is much smaller than the radius of curvature of the surface.

Cylinder. We now consider the exchange of a granular bed with a circular cylinder whose axis is perpendicular to the filtration velocity. In this case the following conditions hold near the surface of the cylinder:

$$u_r \approx 2u(y/R)\cos\theta,\ u_\theta \approx -2u\sin\theta,\ y = r - R,$$
$$D_\perp \approx (1+\gamma\sin\theta)D,\ \gamma = 0.76au/D. \tag{18}$$

The components of the filtration velocity correspond to potential flow around the cylinder; strictly speaking, these components can be found from the linear-filtration problem, i.e., for a bed of sufficiently small particles. The value of the angle $\theta = 0$ of the cylindrical coordinate system corresponds to a line drawn from the center of the cross section of the cylinder to its rear point.

Instead of Eq. (7) in this case we have the following equation in the approximation of a thin diffusion layer:

$$\alpha\frac{\partial T}{\partial t} + 2u\frac{y}{R}\cos\theta\frac{\partial T}{\partial y} - \frac{2u}{R}\sin\theta\frac{\partial T}{\partial\theta} = (1+\gamma\sin\theta)D\frac{\partial^2 T}{\partial y^2} - kT. \tag{19}$$

The boundary conditions in (8) are again imposed on the solution of this equation.

Again introducing the self-similar variable $\eta = y/\delta(t, \theta)$ and the quantity $\varphi = \delta^2$, we find an equation for $T^\circ(\eta)$ which is the same as Eq. (10), whose solution under conditions (8) is given in (11). The equation for φ is

$$\frac{\alpha}{2}\frac{\partial\varphi}{\partial t} - \frac{u}{R}\sin\theta\frac{\partial\varphi}{\partial\theta} = \frac{2u}{R}\varphi\cos\theta + 2(1+\gamma\sin\theta)D. \tag{20}$$

Solving the characteristic system of equations for Eq. (20), we find

$$\varphi\sin^2\theta = \frac{2R^2}{\mathrm{Pe}}\left[\cos\theta - \gamma\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\right] + f\left(\ln\mathrm{tg}\frac{\theta}{2} + \frac{2ut}{\alpha R}\right), \tag{21}$$

where Pe uR/D, and f(z) is an arbitrary function of its argument, which can easily be found fro. the condition $\varphi = 0$ at $t = 0$ by expressing the various functions of θ in (21) in terms of : tg $(\theta/2)$. We finally find

$$\varphi = \frac{2R^2}{\mathrm{Pe}\sin^2\theta}\left\{\cos\theta - \frac{1-\beta^2}{1+\beta^2} - \gamma\left[\frac{1}{2}\theta - \operatorname{arctg}\beta - \frac{1}{4}\sin 2\theta + \frac{\beta(1-\beta^2)}{(1+\beta^2)^2}\right]\right\}, \quad \beta = \exp\left(\frac{2ut}{\alpha R}\right)\operatorname{tg}\frac{\theta}{2}. \quad (22)$$

The local flux of the substance away from the surface of the cylinder is (k = 0)

$$q^\circ = -(1+\gamma\sin\theta) D \left.\frac{\partial T^\circ}{\partial y}\right|_{y=0} = \sqrt{\frac{2\mathrm{Pe}}{\pi}}\frac{T_0 D}{R}\frac{(1+\gamma\sin\theta)\sin\theta}{G(\theta,\ \beta;\ \gamma)},$$

$$G = \left\{\cos\theta - \frac{1-\beta^2}{1+\beta^2} - \gamma\left[\frac{1}{2}\theta - \operatorname{arctg}\beta - \frac{1}{4}\sin 2\theta + \frac{\beta(1-\beta^2)}{(1+\beta^2)^2}\right]\right\}^{1/2}. \quad (23)$$

In the limit $t \to \infty$ ($\beta \to \infty$) we find from this equation the result which has been found previously for the steady-state case without absorption [1, 2]. The characteristic time for relaxation to steady-state exchange at the point with coordinate θ is $(\alpha R/u)|\ln \operatorname{tg}(\theta/2)|^{-1}$; i.e., this time varies from zero for the front and rear points to infinity for the equatorial points.

The total flux away from the cylinder in the case k = 0 is

$$Q^\circ = \sqrt{\frac{2\mathrm{Pe}}{\pi}} T_0 D F\left(\frac{2ut}{\alpha R};\ \gamma\right), \quad F = 2\int_0^\pi \frac{(1+\gamma\sin\theta)\sin\theta}{G(\theta,\ \beta;\ \gamma)}\,d\theta. \quad (24)$$

The local and total fluxes of the substance in the case $k \neq 0$ are found from (23) and (24) by using an equation like (5). In particular, in the steady state we would have

$$q = \sqrt{\frac{2\mathrm{Pe}}{\pi}}\frac{\sigma T_0 D}{R}(1+\gamma\sin\theta)\sin\theta\,\operatorname{tg}^\sigma\frac{\theta}{2}\int_{\operatorname{tg}(\theta/2)}^\infty \frac{d\beta}{\beta^{1+\sigma}G(\theta,\ \beta;\ \gamma)}, \quad (25)$$

$$Q = \sqrt{\frac{2\mathrm{Pe}}{\pi}} T_0 D \int_0^\infty e^{-z} F\left(\frac{z}{\sigma};\ \gamma\right) dz, \quad \sigma = \frac{kR}{2u}. \quad (26)$$

It is not difficult to show that in the limits $\sigma \to 0$ and $\sigma \to \infty$ Eqs. (25) and (26) yield results which have been found previously for the cases of convective diffusion without absorption and from the penetration theory of diffusion from a uniformly accessible surface in the absence of convective transport. In this latter case, however, the effect of convective dispersion on the effective "transverse" diffusion (dispersion) coefficient $D_\perp$ is taken into account.

Sphere. For flow around a sphere in the linear filtration problem we have the following relations near the surface of the sphere:

$$u_r \approx 3u(y/R)\cos\theta, \quad u_\theta \approx -(3/2)u\sin\theta, \quad y = r - R,$$

$$D_\perp \approx (1+\gamma\sin\theta)D, \quad \gamma = 0.57au/D. \quad (27)$$

In place of Eq. (7) or (19) in this case we must solve the following equation under conditions (8):

$$\alpha\frac{\partial T}{\partial t} + 3u\frac{y}{R}\cos\theta\frac{\partial T}{\partial y} - \frac{3u}{2R}\sin\theta\frac{\partial T}{\partial\theta} = (1+\gamma\sin\theta)D\frac{\partial^2 T}{\partial y^2} - kT. \quad (28)$$

Again introducing the variables $\eta = y/\delta(t,\ \theta)$ and $\varphi = \delta^2$, we find (11) for $T^\circ(\eta)$, and for φ we find an equation to replace (20):

$$\frac{\alpha}{2}\frac{\partial\varphi}{\partial t} - \frac{3}{4}\frac{u}{R}\sin\theta\frac{\partial\varphi}{\partial\theta} = \frac{3u}{R}\varphi\cos\theta + 2(1+\gamma\sin\theta)D. \quad (29)$$

From the solution of the characteristic system for this equation we find, in place of (21),

$$\varphi\sin^4\theta = \frac{8R^2}{3\mathrm{Pe}}\left[\cos\theta - \frac{1}{3}\cos^3\theta - \gamma\left(\frac{3}{8}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{32}\sin 4\theta\right)\right] + f\left(\ln\operatorname{tg}\frac{\theta}{2} + \frac{3ut}{2\alpha R}\right), \quad \mathrm{Pe} = \frac{uR}{D}. \quad (30)$$

Proceeding as above, we find, in place of (22),

$$\varphi = \frac{8R^2}{3\mathrm{Pe}\sin^4\theta}\left\{\cos\theta - \frac{1-\beta^2}{1+\beta^2} - \frac{1}{3}\cos^3\theta + \frac{1}{3}\left(\frac{1-\beta^2}{1+\beta^2}\right)^3 - \right.$$

$$-\gamma\left[\frac{3}{8}\,\theta-\frac{3}{4}\,\mathrm{arctg}\,\beta-\frac{1}{4}\,\sin 2\theta+\frac{\beta(1-\beta^2)}{(1+\beta^2)^2}+\frac{1}{32}\sin 4\theta-\right. \tag{31}$$

$$\left.\left.-\frac{1}{4}\,\frac{\beta(1-\beta^2)(1-6\beta^2+\beta^4)}{(1+\beta^2)^4}\right]\right\},\quad \beta=\exp\left(\frac{3ut}{2\alpha R}\right)\mathrm{tg}\,\frac{\theta}{2}.$$

The local flux of the substance away from the spherical surface in the case k = 0 turns out to be

$$q^\circ=-(1+\gamma\sin\theta)\,\frac{\partial T^\circ}{\partial y}\bigg|_{y=0}=\sqrt{\frac{3\mathrm{Pe}}{2\pi}}\,\frac{T_0D}{R}\,\frac{(1+\gamma\sin\theta)\sin^2\theta}{G(\theta,\ \beta;\ \gamma)},$$

$$G=\left\{\cos\theta-\frac{1-\beta^2}{1+\beta^2}-\frac{1}{3}\cos^3\theta+\frac{1}{3}\left(\frac{1-\beta^2}{1+\beta^2}\right)^3-\right.$$

$$-\gamma\left[\frac{3}{8}\,\theta-\frac{3}{4}\,\mathrm{arctg}\,\beta-\frac{1}{4}\sin 2\theta+\frac{\beta(1-\beta^2)}{(1+\beta^2)^2}+\right. \tag{32}$$

$$\left.\left.+\frac{1}{32}\sin 4\theta-\frac{1}{4}\,\frac{\beta(1-\beta^2)(1-6\beta^2+\beta^4)}{(1+\beta^2)^4}\right]\right\}^{1/2}.$$

In the limit $t\to\infty$ ($\beta\to\infty$) we again find the known result for the steady state without absorption [1, 2]. In the case $\gamma = 0$ we find from these equations the equations derived in [11]. The characteristic time for relaxation to the steady state is the same as that for a cylinder.

The total flux away from a sphere in the case k = 0 is given by

$$Q^\circ=\sqrt{6\mathrm{Pe}}\,T_0DRF\left(\frac{3ut}{2\alpha R};\ \gamma\right),\quad F=\int\limits_0^\pi\frac{(1+\gamma\sin\theta)\sin^3\theta}{G(\theta,\ \beta;\ \gamma)}\,d\theta. \tag{33}$$

The local and total fluxes of the substances in the case $k\neq 0$ are found from (32) and (33) and are described by relations reminiscent of (25) and (26):

$$q=\sqrt{\frac{3\mathrm{Pe}}{2\pi}}\,\frac{\sigma T_0D}{R}\,(1+\gamma\sin\theta)\sin^2\theta\,\mathrm{tg}^\sigma\frac{\theta}{2}\int\limits_{\mathrm{tg}(\theta/2)}^\infty\frac{d\beta}{\beta^{1+\sigma}G(\theta,\ \beta;\ \gamma)}, \tag{34}$$

$$Q=\sqrt{6\mathrm{Pe}}\,T_0DR\int\limits_0^\infty e^{-z}F\left(\frac{z}{\sigma};\ \gamma\right)dz,\quad \sigma=\frac{2}{3}\,\frac{kR}{u} \tag{35}$$

(these relations correspond to the steady state).

To determine the fluxes away from a cylinder and a sphere in a filtering granular bed for various values of the parameters we would need to carry out extensive numerical calculations, in contrast with the flux away from a plate. The conclusions reached for the plate, however, remain qualitatively correct for objects of other shapes, as do some of the conclusions which follow from an analysis of the corresponding problems without absorption. An increase in the parameter k leads to a substantial increase in the local and total fluxes; the distribution of the flux over the surface in the flow becomes progressively more uniform. An increase in the importance of convective dispersion in comparison with molecular transport (i.e., an increase in γ) leads to an increase in the local flux, which is particularly marked in the equatorial zone of the object; it also shifts the flux maximum from the front point to a point downstream along the surface.

This theory has certain limitations. First, it ignores the existence of a layer of relatively high porosity near the surface; this layer is important when its thickness becomes comparable to δ, the thickness of the diffusion boundary layer, for example, at very small values of t or at very large values of Pe in the vicinity of the front point of the object. Furthermore, we have ignored the contact surface and also the dispersion associated with fluctuations in the velocity of the continuous phase which result from random fluctuations in the porosity of the bed. Finally, we have used the single-temperature model based on Eq. (1). This model is exact if we are dealing with the mass diffusion of an impurity in the gaps between particles which are impenetrable for the impurity (i.e., if there is no exchange between phases, but the impurity may disappear, of course, in a chemical reaction at the surfaces of the particles or between them). This model is also exact in the case of a very rapid removal of particles from the diffusive boundary layer and in a study of heat transfer in the case in which the particles serve as certain effective heat sinks [5]. In the case of heat transfer in a still or slightly fluidized bed, in contrast, in which we cannot assume a very intense mixing of particles, this model is valid only if the heat transfer by the disperse phase is ignored, in cases in which the transfer differs only slightly from steady-state transfer [7].

NOTATION

a, Particle radius; c_0, c_1 and d_0, d_1, specific heats and densities of the continuous phase and of the particle material, respectively; D_e, $D_\parallel$, $D_\perp$ and D, dispersion tensor, its eigenvalues, and the effective coefficient of molecular thermal diffusivity (or the diffusion coefficient), respectively; F, G, functions introduced in (23), (24) and in (32), (33) for the cylinder and the sphere, respectively; $k_\parallel$, $k_\perp$, coefficients in (3); K, k, absorption coefficients; L, length of the plate; Q, q, total and local fluxes; R, radius of the cylinder or sphere; r, radius; T, T_0, temperature (or concentration) and its value at the surface of the object; t, time; **u**, filtration velocity; x, y, longitudinal and transverse coordinates; α, coefficient defined in (1), (2); β, variable introduced in (22) or (31); γ, structural Pecelt number, defined in (6), (18) or (27); δ, thickness of the diffusion boundary layer; ε, porosity of the bed; $\eta = y/\delta$, self-similar variable; θ, angular variable; $\varkappa$, coefficient in (4); λ_e, effective dispersion tensor, introduced in (1); ξ, ξ_L, dimensionless longitudinal coordinate and length of the plate; σ, parameter introduced in (26) and (35) for the cylinder and the sphere, respectively; $\varphi = \delta^2$; Pe = uR/D; the degree symbol corresponds to the case with k = 0; the subscripts 0 and ∞ denote the fluxes corresponding to the case $k \to 0$ and $k \to \infty$, respectively.

LITERATURE CITED

1. Yu. A. Buevich and D. A. Kazenin, "Heat and mass transfer to objects of various shapes in still and slightly fluidized granular beds," in: Heat and Mass Transfer-5 [in Russian], Vol. 6, Minsk (1976), pp. 19-24.
2. Yu. A. Buevich and D. A. Kazenin, "Limiting case of heat and mass transfer to cylinders and spheres in a filtering granular bed," Zh. Prikl. Mekh. Tekh. Fiz., No. 5, 94-102 (1977).
3. G. Astarita, Mass Transfer with Chemical Reactions, Elsevier, New York (1967).
4. B. I. Brounshtein and G. A. Fishbein, Hydrodynamics and Heat and Mass Transfer in Disperse Systems [in Russian], Khimiya, Leningrad (1977).
5. Yu. A. Buevich, D. A. Kazenin, and N. N. Prokhorenko, "Model for heat transfer of a bed with pronounced fluidization with an immersed surface," Inzh.-Fiz. Zh., 29, No. 3, 410-418 (1975).
6. A. G. Gorelik, V. S. Beskov, N. P. Radkevich, and A. G. Lyubarskii, "Heat transfer in a still catalyst bed," Teor. Osn. Khim. Tekh., 8, No. 3, 394-401 (1974).
7. Yu. A. Buevich and E. B. Perminov, "Time-varying heating of an immobile granular bed," Inzh.-Fiz. Zh., 38, No. 1, 29-37 (1980).
8. P. C. Saffman, "A theory of dispersion in a porous medium," J. Fluid Mech., 6, Part 3, 321-349 (1959).
9. E. N. Lightfoot, "Unsteady diffusion with first-order reaction," AIChE J., 10, Part 2, 278-283 (1964).
10. V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs, New Jersey (1962).
11. E. Ruckenstein, "Mass transfer between a single drop and a continuous phase," Int. J. Heat Mass Transfer, 10, Part 12, 1785-1792 (1967).
12. Yu. A. Buevich and D. A. Kazenin, "An exact solution of the problems of convective heat transfer for longitudinal flow around a plate in a steady-state or fluidized bed," Khim. Mashinostr., 6, 85-87 (1976).